FILTRATION OF A NON-NEWTONIAN FLUID THROUGH A CRACKED-POROUS MEDIUM

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Basic processes of fluid filtration in an elastically compressible cracked-porous material in the presence of an initial filtration gradient are considered.

1. For describing various non-Newtonian systems encountered in the processes of petroleum extraction, the viscous-plastic model has become very common. A characteristic feature of such a fluid is that it starts to move only when the pressure gradient exceeds a certain critical value, called the initial gradient. The corresponding law of filtration was suggested in [1]. Further development of this trend in filtration was the concern of a great number of investigations, the main ones of which are [2-8].

Below we consider filtration of a fluid with an initial gradient in a cracked-porous medium represented by a set of two mutually penetrating continua [8]. Filtration is considered within the framework of the model of [9]. According to this model, the effective characteristics of the filtration process depend greatly on the stressed state of the medium and the liquid pressure in cracks. On the basis of ideas advanced in [1, 9], the equations of one-dimensional flow of fluid with an initial gradient are written in dimensionless form as

$$a\frac{\partial\varphi_1}{\partial t}=\frac{g}{\eta^j}\frac{\partial}{\partial\eta}\left[\eta^j\varphi_1^3\left(\frac{\partial\varphi_1}{\partial\eta}-G_1\right)\right]+m,$$
$$\frac{\partial\varphi_2}{\partial t}=\frac{\varepsilon g}{\eta^j}\frac{\partial}{\partial\eta}\left[\eta^j\left(\frac{\partial\varphi_2}{\partial\eta}-G_2\right)\right]-m, \quad (1)$$

where $\varphi_1\leq 1$, $\varphi_2\leq 1$ are the fluid pressures in cracks and blocks; $\varepsilon \ll 1$ is the ratio of the permeabilities of blocks and cracks at the initial pressure in the bed p^0; j = 0, 1, 2, depending on the flow symmetry. The nonlinear term in Eq. (1) characterizes the elastic deformation of cracks, which, according to [9], can be substantial, even to the extent of their closure. According to this, the parameter a, which has the meaning of the ratio between the elastic capacities of cracks and blocks, can be of any order, in contrast to the model of [8] of a cracked-porous medium, for which $a \ll 1$. The characteristic time for Eqs. (1) is equal to unity. In the case of finite region, $\eta\leq 1$; for an infinite region, the length scale is selected in such a way as to have g = 1 in Eq. (1). It is assumed that the initial gradients for cracks and blocks can be different.

System (1) is meaningful only for $\varphi_1 > 0$ when the dimensional pressure in cracks is higher than a certain critical value σ. When $\varphi_1\leq 0$, the cracks are closed, and in this zone the filtration of fluid occurs through blocks in the elastic regime. On the unknown boundary between the zones, conjugation conditions are fulfilled. Commercial experiments [10] demonstrate that for the wells selected $\sigma \approx (0.9-0.75)p^0$. The mass transfer term m should depend on the pressure drop in cracks and blocks and on the initial filtration gradient in blocks G_2. We can assume with sufficient accuracy that

$$m\sim\frac{\varphi_2-\varphi_1}{l}-G_2,$$

where l is the characteristic dimension of a block. It is known that $l \sim 10^{-2}-10^{-3}$, $G_2 \sim 1-10$; thus, the initial gradient will exert a substantial effect on mass transfer between cracks and blocks when $|\varphi_2-\varphi_1| \sim 10^{-2}-10^{-5}$. In this case

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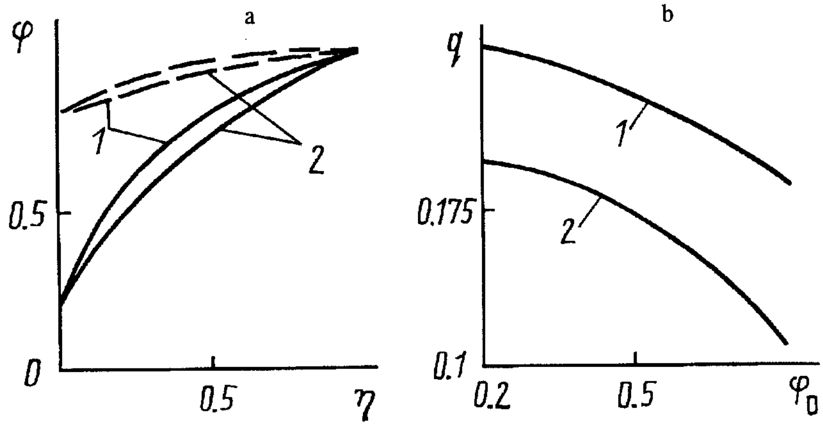

Fig. 1. Pressure in a collector (a) and the discharge $q = q(\varphi_0)$ (b) in a steady-state case, $g = 1$; $G_{1,2} = 0.01$ (1) and 0.1 (2); a) $\varphi_0 = 0.2$ (solid lines) and 0.8 (dashed lines). All the parameters in Figs. 1-4 are dimensionless.

the system of pores-cracks can be regarded as homogeneous and mass transfer between them can be neglected on the whole. Therefore, the mass transfer term can be preserved in Eq. (1) in the standard form [8]: $m=\varphi_2-\varphi_1$.

2. In a steady-state case, neglecting the flow through blocks ($\varepsilon = 0$), we obtain from Eq. (1) the equation for pressure

$$\frac{(\alpha\varphi)^3\eta^j d\varphi}{1+(\alpha\varphi)^3\eta^j} = G_1 d\eta, \quad \alpha = \left(\frac{G_1}{q}\right)^{1/3}, \quad q = \eta^j\varphi_0^3\left[\left(\frac{d\varphi}{d\eta}\right)_0 - G_1\right], \tag{2}$$

where the derivative $(d\varphi/d\eta)_0$ is calculated at the face. For $j = 0$ (filtration to a gallery) the function $\varphi = \varphi(\eta)$, after integration of Eq. (2), is determined in implicit form by

$$q\alpha^4\eta = \delta(\alpha\varphi) - \delta(\alpha\varphi_0),$$
$$\delta(u) = u - \frac{1}{3}\ln(1+u) + \frac{1}{6}\ln(1-u+u^2) - \tag{3}$$
$$- 3^{-1/2}\operatorname{arctg}[3^{-1/2}(2u-1)].$$

An example of calculations with the use of Eq. (3) is shown in Fig. 1. An increase in the initial gradient decreases the discharge of the gallery and the pressure in the collector.

Using Eq. (2) and the boundary conditions $\varphi(0) = \varphi_0$, $\varphi(1) = 1$, we obtain the asymptotics of the process.

When $(\alpha\varphi)^3 \ll 1$, which, considering Eq. (2), means a rather small initial gradient

$$G_1 \ll (\partial\varphi_1/\partial\eta)_0 - G_1,$$

we find

$$\varphi_1^4 \simeq (1-\varphi_0^4)\eta + \varphi_0^4.$$

Consequently, in this case the initial gradient does not influence filtration.

When $(\alpha\varphi_1)^3 \gg 1$, i.e., when the initial gradient is large enough, Eq. (2) yields $\varphi = G_1\eta + \varphi_0$. The steady-state process is possible only when $\varphi_0 < 1 - G_1$. Then the discharge can be expressed as

$$q = 2G_1\varphi_0^2(1-\varphi_0-G_1)/(1-\varphi_0^2). \tag{4}$$

With $\varphi_0 \to (1-G_1)$, the dimension of the perturbation zone $l \to l_* = (1-\varphi_0)/G_1$ and the discharge $q \to 0$. Moreover, $q = 0$ when $\varphi_0 = 0$; consequently, Eq. (4) $q = q(\varphi_0)$ is nonmonotonous. It can be shown that for the pressure in the gallery

$$0 < \varphi_0 = \varphi_*(G_1) =$$
$$= -2\cos\left[\frac{\pi}{3} + \frac{1}{3}\arccos(G_1 - 1)\right] < 1 - G_1 \tag{5}$$

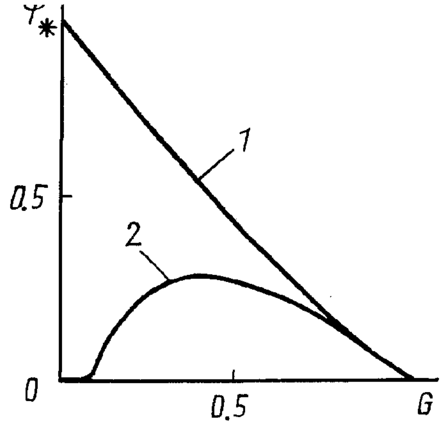

Fig. 2. Optimum seam pressure in the steady-state case (1—Eq. (5); 2—calculation by Eq. (3)) as a function of the initial gradient.

the discharge is maximum.

Figure 2 shows optimum functions $\varphi_*(G_1)$ calculated from Eqs. (5) and (3). When the initial gradient G_1 decreases, the curves differ substantially, which is associated with violation of the condition $(\alpha\varphi_1)^3 >> 1$. We note, however, that at small enough values of φ_0 it is necessary to take into account the flow through blocks and this will change the form of the left part of curve 2.

3. In the case of unsteady filtration, we consider approximate solutions of Eqs. (1). There is no information in the literature concerning the relationship between the initial gradients G_1 and G_2. Physically, the most real case is that when $G_2 > G_1$. Taking into account this uncertainty, we will consider below the general case of different values of the initial gradients G_1 and G_2.

At a prescribed pressure φ_0 on the gallery we have

$$\varphi_i = \varphi_0 + \beta_i\eta/l_i + \gamma_i\eta^2/l_i, \quad 0 \leqslant \eta \leqslant l_i(t),$$
$$\beta_i = 2(1-\varphi_0) - G_i l_i, \quad \gamma_i = \varphi_0 - 1 + G_i l_i \quad (i = 1,\ 2), \tag{6}$$

where $l_i(t)$ are perturbation fronts propagating through cracks and blocks. At $\eta = l_i$, the conditions $\varphi_i = 1$, $\partial\varphi_1/\partial\eta = G_1$ are fulfilled.

The functions $l_i = l_i(t)$ are found from the second integral relations corresponding to Eqs. (1):

$$a\frac{d}{dt}(h_1 l_1^2) = F_1 + f, \quad \frac{d}{dt}(h_2 l_2^2) = F_2 - f,$$
$$F_1 = g\left\{\frac{1}{4}(\varphi_0^4 - 1) + G_1 l_1\left[\varphi_0^3 + \left(\frac{3}{2}\beta_1^3 + \gamma_1\right)\varphi_0^2 + \right.\right.$$
$$\left.\left. + \left(\beta_1^2 + \frac{3}{5}\gamma_1^2\right)\varphi_0 + \frac{1}{4}\beta_1^3 + \frac{3}{5}\beta_1^2\gamma_1 + \frac{1}{2}\beta_1\gamma_1^2 + \frac{1}{7}\gamma_1^3\right]\right\}, \tag{7}$$
$$F_2 = \varepsilon g(G_2 l_2 - 1 + \varphi_0), \quad h_i = (\varphi_0 - 1 - G_i l_i)/12,$$
$$f = (l_1^2 - l_2^2)/2 + (\varphi_0/2 + \beta_2/3 + \gamma_2/4)\, l_2^2 -$$
$$- (\varphi_0/2 + \beta_1/3 + \gamma_1/4)\, l_1^2.$$

Taking into account Eq. (6), we determine the discharge

$$q = \left[\varphi_1^3\left(\frac{\partial\varphi_1}{\partial\eta} - G_1\right) + \varepsilon\left(\frac{\partial\varphi_2}{\partial\eta} - G_2\right)\right]_0 =$$
$$= 2\varphi_0^3\left(\frac{1-\varphi_0}{l_1} - G_1\right) + 2\varepsilon\left(\frac{1-\varphi_0}{l_2} - G_2\right). \tag{8}$$

In the linear case, when cracks are deformed slightly (the model of [8]), we should assume that $\varphi_1^3=1$ in Eqs. (1) and, correspondingly, that $\varphi_0^3=1$ in Eq. (8). The first term in Eq. (8) is the discharge of the gallery in a cracked bed. For brevity, we designate the media as LM (linear medium), CM (cracked medium) and NM (nonlinear medium).

On the basis of Eqs. (6)-(8) we can conclude that the main conclusions derived in [2, 4] concerning fluid filtration with an initial gradient in a porous collector are applicable to all the above-indicated media: NM, LM and CM.

Actually, from Eqs. (6)-(8) it follows that at small times when

$$l_i \ll l_* = \min(l_{1*},\ l_{2*}),\ l_{i*} = (1-\varphi_0)/G_i \quad (i=1,\ 2),$$

the initial gradient does not influence the process. When $l_i \to l_{i*}$ the fluid flow through the i-th phase q_i tends to zero, and pressure distribution (6) tends asymptotically, $t\to\infty$ to the limiting one

$$\varphi_{i*} = \varphi_0 + (1-\varphi_0)\,\eta/l_{i*}. \tag{9}$$

We consider in more detail the indicated asymptotics.

In the case when $l_i \ll l_*$, Eq. (7) has the solution

$$l_1^2 = u + w,\quad l_2^2 = -au + w,$$
$$u = (d_1 - ad_2)(1-\exp(st))/(1+a)^2,\ d_2 = 12\varepsilon g, \tag{10}$$
$$w = (d_1+d_2)t/(1+a),\ d_1 = 3g(1-\varphi_0)^4(1-\varphi_0)^{-1},\ s = -a^{-1}(1+a).$$

We elucidate the character of the fall of the discharge in an NM at small times as a function of the parameter a. From Eqs. (8) and (10) with a << 1 (rapid filtration [11]) we find that

$$q \approx \psi(\varphi_0)[1+t-\exp(-t/a)]^{-1/2},$$
$$\psi(\varphi_0) = 2\varphi_0^3(1-\varphi_0)^{3/2}[3g(1-\varphi_0^4)]^{-1/2}, \tag{11}$$

and for a >> 1

$$q \approx \psi(\varphi_0)(a/t)^{1/2}. \tag{12}$$

As the parameter a increases, filtration slows down [11]. When a >> 1, filtration is rather slow, so that cracks cease to play the part of basic channels and the collector acts as homogeneous. Therefore, the character of Eq. (12) $q \sim t^{-1/2}$ is the same as for a porous collector.

Comparing Eqs. (11) and (12) we can find that at small times the discharge will fall more rapidly when a<< 1.

The function $\psi(\varphi_0)$ has a maximum at $\varphi_0 \approx 3/4$. This agrees with the results of steady-state filtration of a Newtonian fluid [12].

The asymptotic character of the process with $l_i \to l_{i*}$ can be ascertained in the following way. We sum up Eqs. (7) and write the result in the form

$$\frac{d}{dt}(ah_1l_1^2 + h_2l_2^2) = -\int_0^{l_1} q_1 d\eta - \int_0^{l_2} q_2 d\eta. \tag{13}$$

When $l_i \to l_{i*}$, it follows from Eq. (7) that $h_1, h_2 \to 2(\varphi_0 - 1)$, while the right-hand side of Eq. (13) tends to zero. From this it follows that when $l_i \to l_{i*}$, the time $t\to\infty$.

The limiting overall extraction of fluid in the case of NM is determined from the balance condition

$$Q_{NM} \approx \overset{\circ}{m}_1 \int_0^{l_{1*}} (1-\varphi_{1*})(1+\nu_1\varphi_{1*})d\eta + \overset{\circ}{m}_2\nu_2\int_0^{l_{2*}}(1-\varphi_{2*})d\eta, \tag{14}$$

$$\nu_1 = (p^\circ - \sigma) K_\rho^{-1}, \quad \nu_2 = (p^\circ - \sigma)(K_\rho^{-1} + K_m^{-1}).$$

In reality, $p^o - \sigma \sim 1$—10 MPa, K_ρ, $K_m \sim 10^2$-10^3 MPa; consequently, $\nu_1 \ll 1$, $\nu_2 \ll 1$. Since usually for cracked-porous collectors $m^o_1 \ll m^o_2$, then the term involving the coefficient ν_1 in Eq. (15) can be neglected. Taking into account Eq. (9) we find

$$Q_{NM} \approx \frac{(1-\varphi_0)^2}{2} \left(\frac{\overset{\circ}{m}_1}{G_1} + \frac{\overset{\circ}{m}_2 \nu_2}{G_2} \right).$$

Assuming for LM that cracks deform in the same way as the pores of blocks, we obtain

$$Q_{LM} \approx \frac{(1-\varphi_0)^2}{2} \left(\frac{\overset{\circ}{m}_1 \nu_1}{G_1} + \frac{\overset{\circ}{m}_2 \nu_2}{G_2} \right).$$

Comparing the fluid volumes extracted from NM and LM, we find that the expression

$$Q_{NM} - Q_{LM} = \frac{(1-\varphi_0)^2 (1-\nu_1) \overset{\circ}{m}_1}{2G_1} > 0$$

is always positive, since $|\nu_1| < 1$.

In the case of CM, the extraction $Q_{CM} = m^o_1(1-\varphi_0)^2/2G_1 < Q_{NM}$.

The dimension of the draining zone in cracks $l_{1*} = (1-\varphi_0)/G_1$ is the same for all the media: NM, LM, CM. However, the speed of propagation of a perturbation for finite values of t in the media indicated will naturally be different, as confirmed by numerical calculations. Similarly to the case of a Newtonian fluid [11], the greatest speed of propagation of a perturbation is in LM, and the least in CM.

Let us consider putting into operation a well with the constant discharge q. The pressure profiles are approximated by the expressions

$$\varphi_i = 1 + \beta_i - G_i l_i + \beta_i \ln(\eta/l_i) + (G_i l_i - \beta_i)\eta/l_i, \tag{15}$$

where the coefficients β_i are determined from the conditions in the well

$$\eta = \eta_0, \; q = \eta_0 (\varphi_1^3 \partial\varphi_1/\partial\eta + \varepsilon \partial\varphi_2/\partial\eta), \; \varphi_1 = \varphi_2. \tag{16}$$

Equations for the perturbation fronts $l_i = l_i(t)$, with the flow through the blocks being neglected, are written in the following form:

$$a d\delta_1/dt = -\delta_1 + \delta_2 + 12gq, \; d\delta_2/dt = \delta_1 - \delta_2, \; \delta_i = (\beta_i + 2G_i l_i) l_i^2, \tag{17}$$

and they have the solution

$$\delta_1(t) = c\left(t + \frac{1+\exp(st)}{1+a}\right),$$
$$\delta_2(t) = c\left(t - \frac{a(1-\exp(st))}{1+a}\right), \quad c = 12gq(1+a)^{-1}. \tag{18}$$

We obtain the asymptotics of the process of pressure fall in the well. At small times $t \ll t_*$, when

$$G_1 l_1 \ll \beta_1, \tag{19}$$

the initial gradient does not influence filtration, and the pressure in the well is determined in implicit form by the expression

$$\varphi_0 \approx 1 + \beta_1(1 + \ln(\eta_0/l_1)), \quad l_1 \approx (\delta_1/\beta_1)^{1/2}, \quad \beta_1 \approx q/\varphi_0^3, \tag{20}$$

where the functions $l_1(t)$, $\beta_1(t)$ are found from Eqs. (16) and (17).

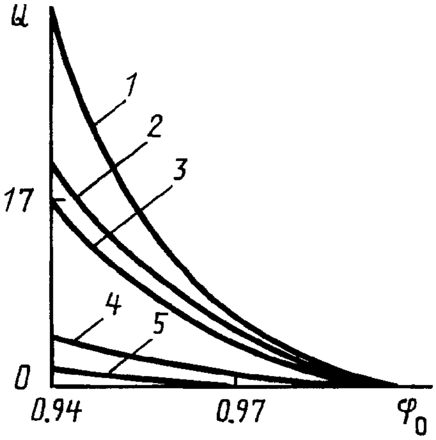

Fig. 3. Overall extraction of fluid, g = 10; $G_{1,2} = 0.1$; a = 10: NM (1); CM (2), LM (3); a = 1: NM (4); G = 0.2, a = 1: NM (5).

Practical calculations of the asymptotics are fairly simple and are performed as follows. On each time step t (t grows from zero) the value of the function $\delta_1(t)$ is determined from Eq. (18), and then the pressure in the well $\varphi_0(t)$ is found from Eq. (20); thereafter condition (19) is verified. If this condition is complied with, the calculation is continued.

At large enough times $t \gg t_*$, when $G_1 l_1 \gg \beta_1$, the pressure in the well varies according to the law

$$\varphi_0 \approx 1 - G_1 l_1 + \beta_1 \ln(\eta_0/l_1), \quad l_1^3 \approx \delta_1 (2G_1)^{-1}, \quad \beta_1 \approx q/\varphi_0^3 - G_1\eta_0. \tag{21}$$

From Eq. (18) it follows that with growth of time the collector operates as homogeneous, since $\delta_1 \approx \delta_2 \approx ct$; therefore Eq. (22) can be represented as

$$\varphi_0 \approx 1 - G_1 l_1 + (q/\varphi_0^3 - G_1\eta_0) \ln(\eta_0/l_1), \quad l_1^3 = ct(2G_1)^{-1}.$$

Consequently, at large times $\varphi_0 \sim (G_1^2 qt)^{1/3}$, which coincides with the results for a porous collector [2, 4].

The time instant $t = t_*$ satisfies the equation

$$q\varphi_0^{-3}(t_*) \approx [G_1^2 \delta_1(t_*)]^{1/3}.$$

Examples of numerical calculations with use of system (1) and the above-indicated asymptotics are given in Figs. 3 (filtration toward the gallery) and 4 (filtration toward the well). It is adopted in the calculations that $\varepsilon = 10^{-2}$. The value of the dimensionless initial gradient G was selected in the following way. In Eq. (1), $G = G_0 M/(p^0 - \sigma)$, where M is the characteristic dimension of the system; G_0 is the dimensional initial gradient. According to [4], under the bed conditions the value $G_0 \sim 10^3$-10^4 N/m is possible. Practically, $p^0 - \sigma \sim 1$—10 MPa, $M \sim 10^2$-10^3 m; consequently in Eq. (1) $G \sim 10^{-2}$-10.

The greatest overall volume of the fluid (Fig. 3) is extracted from NM and the smallest from LM. The overall extraction from NM increases with growth of the ratio a between the elastic capacities of cracks and blocks and with decrease in the initial gradient.

As the discharge and the initial gradient grow, the pressure falls more rapidly in the well and correspondingly the time from the start of the process to the closure of cracks decreases (Fig. 4). When the pressure in the well decreases to a certain value, further closure of cracks occurs almost instantly, and a "two-layer" character of the curves is clearly seen. For small enough values of q, G, the process of filtration stabilizes with time, and no closure of cracks takes place (curve 1). As follows from Fig. 4 b, there is also no closure of cracks when fluid is extracted with the initial gradient $G_{1,2} = 0.01$ and with the discharge $q \lesssim 0.046$.

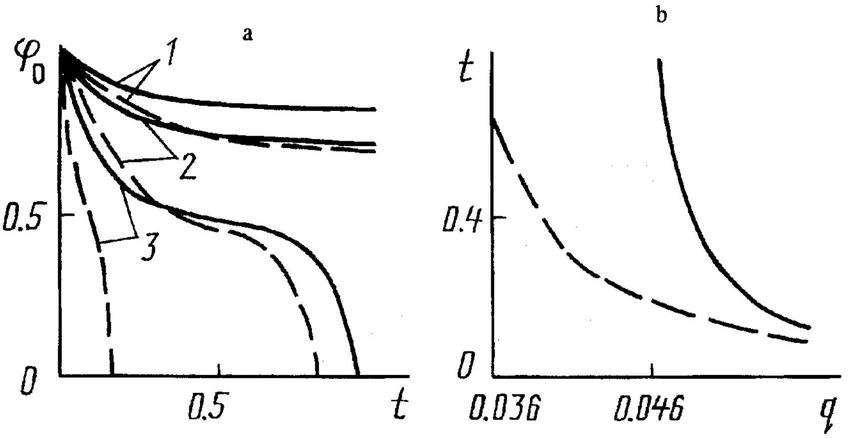

Fig. 4. Pressure drop in the well (a): q = 0.021 (1), 0.036 (2), 0.043 (3) and time of closure of cracks (b): a = g = 10, $G_{1,2} = 0.01$ (solid lines) and 0.1 (dashed lines).

NOTATION

φ, η, t, dimensionless pressure, coordinate, time; η_0, well radius; p, p^o, dimensional pressure and its initial value in the bed; σ, critical pressure; a, ratio of elastic capacities of cracks and blocks; ε, ratio of permeabilities of cracks and blocks under pressure; g, parameter of system (1); G, G_0, dimensionless and dimensional initial gradient; q, Q, discharge and overall extraction of fluid; φ_0, pressure in a well; l, dimension of a perturbation zone; M, characteristic dimension of a system; m, porosity; K_ρ, K_m, elastic constants of fluid and blocks. Subscripts: 1, 2, refer to cracks and blocks, respectively.

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